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MISCELLANEOUS PAPER M-68-6

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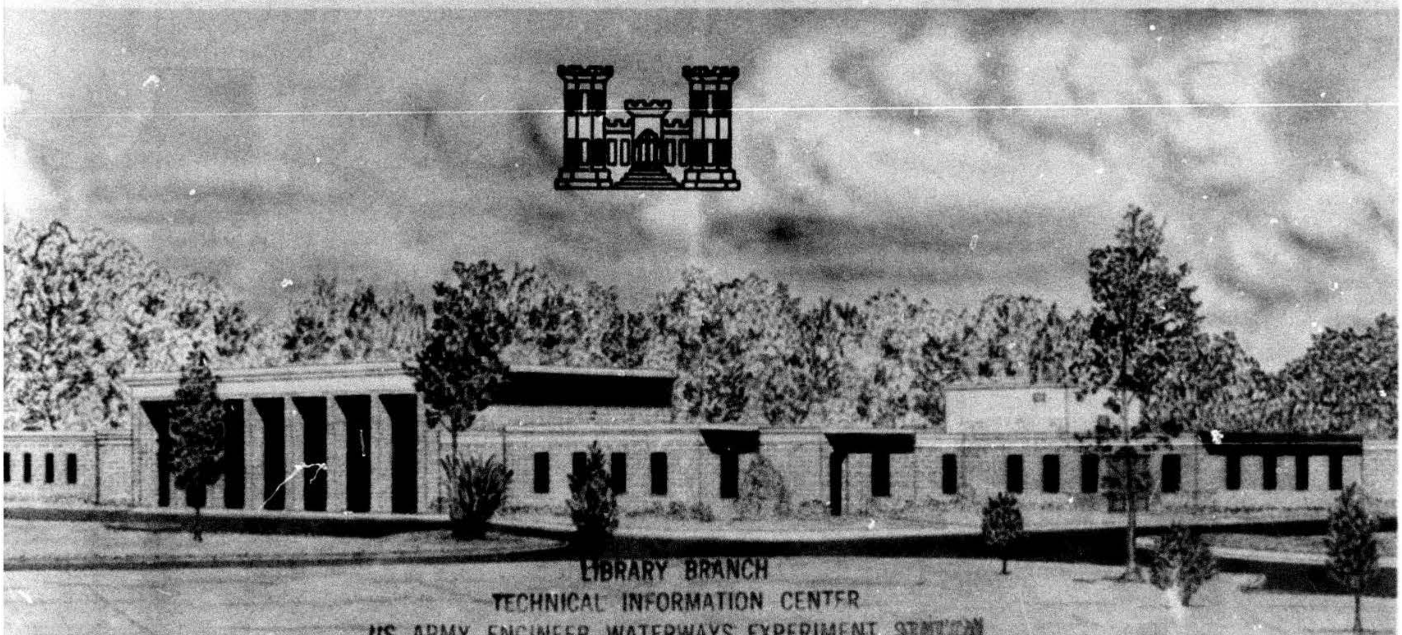
by

K. W. Wiendieck



December 1968

Sponsored by U. S. Army Materiel Command

Conducted by U. S. Army Engineer Waterways Experiment Station, Vicksburg, Mississippi

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Foreword

The study reported herein was performed by Dr. Klaus W. Wiendieck of the Mobility Research Branch (MRB), Mobility and Environmental (M&E) Division, at the U. S. Army Engineer Waterways Experiment Station (WES) as part of the vehicle mobility research program under DA Project 1T0-14501B52A, "Research in Earth Sciences," Task 01, "Terrain Analysis," under the sponsorship and guidance of the Development Directorate, U. S. Army Materiel Command.

The study was conducted during the period January-December 1966 under the general supervision of Messrs. W. J. Turnbull, W. G. Shockley, and S. J. Knight; and under the direct supervision of Dr. D. R. Freitag, Chief of the MRB, and Mr. A. J. Green, Chief, Vehicle Dynamics Section, MRB. This paper was prepared by Dr. Wiendieck.

COL John R. Oswalt, Jr., CE, was Director of WES during this study, and Mr. J. B. Tiffany was Technical Director.

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STRESS-DISPLACEMENT RELATIONS AND TERRAIN-VEHICLE MECHANICS: A CRITICAL DISCUSSION

K. W. WIENDIECK*

INTRODUCTION

THE FORCES that move an earthbound vehicle over the ground are, in the last analysis, soil reaction forces. The engine serves only to generate these soil reactions by transmitting a certain mechanical energy to the running gear. Recognition of this fact makes the interaction between the soil and the running gear the key problem of theoretical soil–vehicle mechanics.

Current theoretical approaches to this fundamental problem are based essentially on the idea of stress–displacement relations for soils. The most commonly used are the pressure–sinkage relation expressed by the equation

$$\sigma = \left(\frac{k_c}{b} + k_\phi\right) z^n \tag{1}$$

and the shear stress–displacement equation

$$\tau = (c + \sigma \tan \phi)\,[1 - \exp(-j/k)] = \tau_{max}\,[1 - \exp(-j/k)]. \tag{2}$$

Both expressions have been proposed by Bekker [1], the latter in a somewhat more complex form to account for peak shear stresses. Equation 2 as given here is a modification introduced by Janosi and Hanamoto [2].

Although it is obvious from the manner of derivation that these equations are no more than convenient mathematical formulations of experimental curves obtained under special conditions, certain researchers seem to consider them as laws of the material. Reference to these equations as "stress–strain relations for soils" or to some of the parameters as "soil constants" has encouraged this tendency.

It must be emphasized that specific stress–strain relations for soil are not yet established [3]. Accordingly, these relations (equations 1 and 2) or similar ones are not in use in the much older and more advanced science of soil mechanics, despite the fact that knowledge of the stress–strain characteristics for soils is highly desirable for many soil mechanics problems. It was not until very recently that first steps were undertaken by soil mechanics researchers to develop theoretically founded stress–strain relations for soils, in the sense of a constitutive law of the

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material, because of the formidable difficulties inherent in such investigations. Further discussion and a list of pertinent literature can be found in the references [4, 5].

Because of the fundamental nature of the problem involved and because of the newness of the concept, one would expect stress–strain relations for soils to be considered with an adequate reserve. However, the author has the impression that these relations–or their basic ideas–were accepted immediately by most of the research workers in the field of terrain–vehicle mechanics. Most of the literature on this subject is related to secondary issues such as curve-fitting techniques, form and size of test equipment, establishment of a definite set of "soil constants", and more or less far reaching modifications of the analytical expressions. The main issue—whether these relations are at all applicable in the field of soil–vehicle mechanics—has rarely been questioned seriously.

This paper deals, first, with pressure–sinkage and shear stress–displacement relations in general terms. Then the particular problem of the applicability of the shear stress–displacement relation (equation 2) to rigid wheels in sand is examined. An approximate relation between the M/RW value and the τ/σ ratio is developed to allow Sela's theory [6] on the shear-to-normal stress ratio beneath rigid wheels on sand to be checked experimentally. This theory is based exclusively on the shear stress–displacement relation and thus provides an excellent opportunity to check the concept as a whole.

STATE OF STRESS

Although no genuinely scientific law has been found to describe the stress–strain relations for soils over the whole stress range, there is little doubt that, in a qualitative sense, these relations are of the form shown by the solid line in Fig. 1. The curve with a peak (dashed line) illustrates a case in which the deformations are

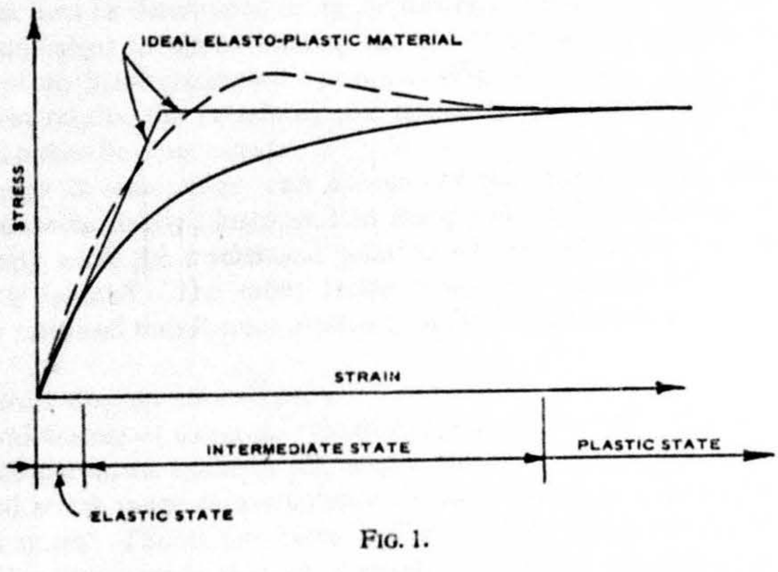

FIG. 1.

accompanied by a recovery of compaction energy. The hypothetical curve for an ideal elastic-plastic material is also shown.

Despite the continuous character of these curves, three different states of stress

can be defined, the limits of which are necessarily somewhat arbitrary. These three states are:

(a) *An elastic state* in which the deformations of the soil can be assumed to stem almost exclusively from the elastic deformations of the particles. The structure of the solid mass as a discontinuous random aggregate of particles remains unchanged.

(b) *An intermediate state* characterized by an increasing number of local grain-to-grain slidings. Although these are systematic in view of the ultimate failure, they do not form a coherent pattern of rupture surfaces. The structure is in permanent rearrangement as the stresses increase, but it remains stable for a given stress.

(c) *A state of actual plastic flow* that follows a definite coherent pattern of rupture surfaces if the geometrical and load conditions are not altered by the flow process itself. The structure of the aggregate is unstable.

These definitions are tentative and may not account for all complexities, especially if interparticle cohesion forces are involved; they are suggested here to emphasize that the states of stress are governed by three different physical processes.

Since the mechanical behavior of soils is rather complex, it might be useful to consider how the related science of soil mechanics deals with these problems.

In classical soil mechanics, generally only two states of stress are considered, the elastic one and the plastic one. This simplification of the rather complex soil behavior is possible because soil mechanics primarily deals with stability problems, i.e. ascertaining that the soil which constitutes or supports a structure will not fail. For this purpose, the soil can be reasonably assumed to behave like an ideal plastic material, and the failure loads evaluated in terms of plasticity. This failure load is then reduced by a safety factor in the order of magnitude between 2 and 3, so that the actual working stresses within the soil fall in a domain where the material can be reasonably considered as elastic. The deformations under varying actual loads can then be determined using the theory of elasticity. This procedure appears to be equivalent to approximating the actual stress–strain characteristics of soils to that of an ideal elastoplastic material (Fig. 1). From an appropriately chosen set of test results, the pertaining soil constants of elasticity and plasticity, respectively, can then be determined.

Although oversimplified, this account of the computational procedure used in soil mechanics clearly illustrates that many soil mechanics problems can be solved adequately while the mechanical behavior of the soil in the "intermediate state of stress" is ignored. The safety factor covers the unexplored region of the intermediate state and thus focuses attention on the extreme ends of the curves.

Shear stress–displacement relation

To which state of stress do equations (1) and (2) refer? Consider first the shear stress–displacement equation (2), which obviously constitutes an attempt to cover the total stress range in a continuous manner, without breaking the relation into distinct states. This is illustrated in Fig. 2 which is reproduced from Wills paper [12]. One argument in support of equation (2) is that the safety factor is irrelevant in terrain–vehicle mechanics and, therefore, the complete shear stress–displacement curve must be considered.

A close examination of how equation (2) has been commonly used reveals, how-

ever, that in practice, this equation refers almost exclusively to the intermediate state of stress. According to Bekker [1], the soil distortion j is expressed by the product of slip s and the distance x, measured from the beginning of the contact area to the point where the shear stress is considered: $j = xs$. Hence, the sequential

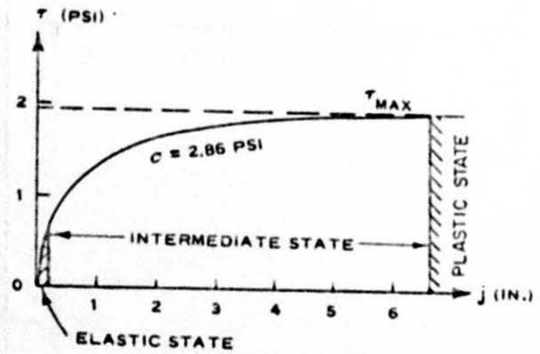

FIG. 2.

consideration of points more and more distant from the leading edge of the contact area of a running gear is equivalent to proceeding by appropriately scaled steps along the horizontal axis, j (Fig. 2). In doing so, the elastic region is left quickly and the region of the intermediate state of stress entered. Because of the exponential nature of the curve, the region of the intermediate state can be left, and the region of the perfect plastic state approached only after proceeding rather far along the distortion axis.* In practice, this occurs only for very high slip values and long contact areas. Indeed, Sela's theoretical work [6] shows that, for a wheel of 51-cm dia., the perfect plastic region is not approached below 30 per cent slip. The use of equation (2), therefore, implies that nearly all of the soil adjacent to the contact area is in the intermediate state of stress, which, thus, is considered characteristic for the interaction of the soil and the running gear. In the light of this statement, it appears justified to question whether the soil in the immediate vicinity of the running gear is, in fact, almost exclusively in the intermediate state, i.e. in a nonplastic, nonelastic condition. The author is convinced that any vehicle producing a noticeable sinkage loads the ground to its ultimate bearing capacity, which indicates that the directly supporting soil is in a state of plasticity. To ignore the perfect plasticity, as suggested by equation (2), therefore appears to be a serious error of principle.

Recent investigations of rigid wheels in cohesionless materials [7, 8] have revealed an additional phenomenon—a certain soil mass apparently attached to the wheel and moving around the instantaneous center of rotation (Fig. 3). Because of the crescent moon-shaped outline of this soil mass, it is suggested that it be called a "lunule". The individual soil particles that form the lunule change constantly, of course, as the wheel moves forward, but the instantaneous existing lunule seems to behave like a solid body [4].

Some controversy may emerge concerning the state of stress of the lunule and of

*According to equation (2), the state of perfect plasticity is never reached for finite displacements.

its geometrical boundary. Considering the lunule as a solid body, and in view of its kinematical function [4], it probably is in an elastic state of stress and bounded by a circle. Wong and Reece [8] refer to the lunule as being in an active plastic state and bounded by a logarithmic spiral.

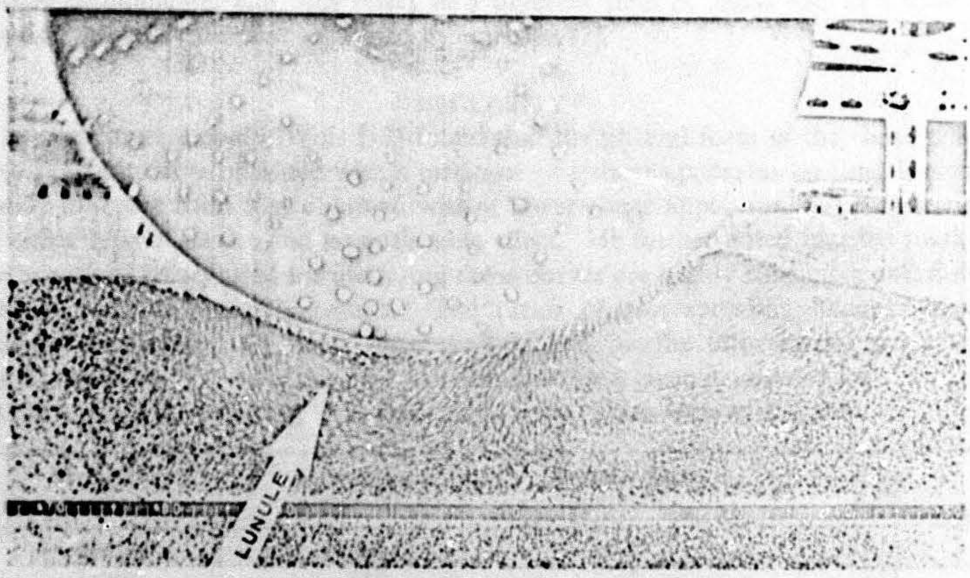

Fig. 3.

Whatever is true, it seems to be definite that the lunule is not in the intermediate state. Therefore, the cohesionless soil immediately beneath a rigid wheel is either totally in a perfect plastic state or partially so, with the remaining part in an elastic state. The shear stress–displacement equation (2) refers to a state of stress that does not exist in the immediate neighborhood of the wheel–soil interface in sand and is thus irrelevant to the wheel–soil interaction. This is likely to hold true also for running gears other than rigid wheels and for soils other than sand. Therefore, the new soil constants introduced by equations (1) and (2) to characterize the stress–strain relation of soils are not only superfluous in terrain–vehicle mechanics, but also misleading.

Pressure–sinkage relation

It is obvious that when a plate is pushed vertically into the soil, the resisting forces are due to a purely plastic phenomenon once a certain entry effect is overcome. Although it is recognized that additional difficulties arise from the compressibility of the material, the main problem is still to determine the failure pattern, for which, in addition to geometrical parameters, only the specific weight γ and the constants of the plasticity c and ϕ are necessary.

This simple fact has been totally obscured by early researchers in the field of terrain–vehicle mechanics. Bekker states that the knowledge of c and ϕ does not suffice to solve the problem. "To determine sinkage, slippage, etc., one must know the corresponding stress–strain relations. To this end, other values are needed" [1].

It is to the credit of the English school of Newcastle-upon-Tyne that the pressure–

sinkage problem has been placed in its proper perspective [9–11]. Studies conducted there show clearly that the empirical parameters k_ϕ and k_c and n of equation (1) cannot be understood as constants of a stress–strain relation, but must be considered as a combination of geometrical parameters and constants of plasticity in the same way as the well-known bearing capacity factors. Hence, equation (1) describes a plastic phenomenon and thus refers to a different state of stress and to a different physical process from that expressed by equation (2).

SIMILARITY

In comparative study, Wills [12] found that the general form of the shear stress–displacement curve obtained with a torsional ring-shear apparatus on sand is significantly different from that obtained with a linear shear apparatus, but that the size of either type of device had no noticeable effect. He further noted that the methods (he mentions three) used for analyzing these curves are highly subjective and do not always yield comparable results. The ratios of corresponding "horizontal soil deformation moduli k" varied from 1:2 to 2:1 for the information given; also, the absolute magnitude of the modulus obtained for a given device depended strongly on the applied curve-fitting technique. The values given were as follows:

Method of analysis	Linear shear apparatus, k_l	Annular shear apparatus, k_a	Ratio k_l/k_a
Bekker	3/4	$1\frac{1}{2}$	1:2
Janosi	1/2	$\frac{1}{4}$	2:1
Adams	3/4	$1\frac{1}{2}$	1:2

These contradictory results are remarkable in that both tests represent pure, although slightly different, sliding processes.

Thus, experimental evidence emphasizes that the shear stress–displacement curves must be considered as soil responses to a particular test device. This is also true of the pressure–sinkage relation (equation 1) in which the parameters k_ϕ, k_c, and n depend on form and size of the plates as reported by Wills [10]. To what extent such test curves represent the soil behavior beneath a wheel or a track remains a conjecture. There is no similarity between the physical process that takes place in the sand beneath a rigid wheel as shown in Fig. 3 and the soil behavior beneath sliding or punching plates.

The author does not share the opinion that refining the mathematical forms of equations (1) and (2) would make them more meaningful for soil–vehicle mechanics. Even if equations could be found with parameters independent of the form and size of the test device, they would characterize soil responses to pure punching or sliding processes only. Whether such isolated "pure" soil responses can be superposed to represent the combined slide-punch action that takes place beneath an actual running gear is the next question to be investigated.

THE PRINCIPLE OF SUPERPOSITION

The principle of superposition is essential for the rational solution of many design problems; it can be applied only if certain basic theoretical requirements are met. For example, load-deformation problems can be solved by means of superposition only if the stress–strain relation is linear. Within the theory of plasticity, partial

solutions can be superposed only if they refer to the same rupture pattern. Therefore, Terzaghi's simple formula for the bearing capacity of shallow strip foundations

$$p = cN_c + qN_q + \frac{1}{2}\ \gamma B N_\gamma$$

is not a rigorous solution since the computation of the N_c and N_q factors is based upon a rupture pattern other than the one used to compute the N_γ factor [13]. However, because the patterns are very similar, the formula can be accepted for engineering purposes.

Pressure-sinkage and shear stress-displacement relations are used to describe, in an isolated manner, two different aspects of soil reactions. Thus, their joint application to the problem of soil-running gear interaction implies, tacitly, the principle of superposition.

The two relations refer to different states of stress–the plastic and the intermediate one; this alone would impede superposition. Even if one neglects this point and assumes that both relations refer to the same state of stress, whether plastic or intermediate, the principle of superposition does not apply. In the intermediate state, the stress-strain relation is certainly not linear; in the plastic state, the rupture patterns resulting from a pure sliding action and a pure punching action are totally different. Therefore, the validity of the principle of superposition must be rejected in this case.

BRIEF OUTLINE OF SELA'S THEORY

Sela [6] made extensive use of the shear stress-displacement relation (equation 2) to describe theoretically the variation of the shear to normal stress ratio along the wheel-soil interface for rigid wheels in sand. As far as this particular problem is concerned, Sela's theory is the most advanced development, and thus provides an excellent opportunity to check the shear stress-displacement relation concept as a whole.

For cohesionless soils ($c = 0$), equation (2) may be transformed into:

$$t = \frac{\tau}{\sigma} = \tan\phi\,[1 - \exp(-j/k)]. \tag{3}$$

Replacing the displacement j by the product of slippage and the distance from the leading edge, one obtains

$$t = \frac{\tau}{\sigma} = \tan\phi\,\{1 - \exp[-R(\alpha - \delta)s/k]\}. \tag{4}$$

Sela [6] refined equation (4) by taking into account the effect of the bow wave, but neglecting the lateral flow. For the bow wave zone, he proposed the equation

$$t = \frac{\tau}{\sigma} = \tan\ \phi\,(1 - \exp\{-R\,[(\alpha - \delta)\,s + (\delta - \alpha_d)\,m]/k\}) \tag{5}$$

where the angle α_d designates the separation point between the bow wave region (as defined by Sela) and the rear part of the interface, and where m is a factor of

proportionality. According to Sela, the original equation (4) remains valid for the rearward trailing zone. Sela defines the angle α_d by the relation

$$\tan \alpha_d = \frac{\tau}{\sigma} = \tan \phi \{1 - \exp[-R(\alpha_d - \delta)s/k]\}.$$

According to Sela's theory, τ/σ increases generally toward both ends of the wheel-soil interface, with the lowest τ/σ value at the point of separation between the forward bulldozing and the rearward trailing zone.

By applying Sela's theoretical concept to wheel tests conducted at the WES, nine different t (i.e. τ/σ) distributions were obtained based on the following numerical data (Fig. 4):

$$R = \frac{71}{2} = 35{\cdot}5 \text{ cm}$$

$$k = 2{\cdot}54 \text{ cm}$$

$$m = 0{\cdot}5$$

$$\tan \phi = 0{\cdot}5$$

$$\text{slip} = 3,\ 14 \text{ and } 33 \text{ per cent.}$$

k and m were not measured for this sand, since their exact determination is highly subjective [6, 12]. The values used by Sela were $k = 2{\cdot}54$ cm and $m = 0{\cdot}65$, but he pointed out that this value of m was too high; therefore, a slightly smaller value, 0·5, was used in this analysis. The value of $\tan \phi = 0{\cdot}5$ was chosen as a convenient reference. The actual friction coefficients of the two tested sand conditions were 0·6 and 0·65, respectively, so the t values (Fig. 4) had to be multiplied by a factor of 1·2 and 1·3, respectively, for comparison with actual tests.

METHOD TO CHECK τ/σ THEORIES

It has been reported elsewhere [4] that accurate measurement of the shear to normal stress ratio by means of pressure transducers incorporated in the rolling surface of the wheel is not easily done. This was recognized by Sela himself, who notes that, especially at the extremes of the wheel-soil interface, "readings of the shear and normal stresses are low and thus a small error in reading causes a large error in the τ/σ value". In tests conducted at the WES [4], scatter in the τ/σ ratio was more than 50 per cent with respect to the mean value. A new method to check any τ/σ theory was developed which is less cumbersome in application, less expensive in use, and has an error margin of only ± 25 per cent. This method is based on the consideration of the mean τ/σ value (t_m) taken over the whole wheel-soil interface, rather than on the variation of τ/σ along the contact length. It can be shown that t_m is roughly equal to M/RW. Since t_m is obtained readily from any τ/σ theory (by graphical construction if necessary) and since M/RW is easily and accurately measureable for a rigid wheel, the relation between the two quantities obviously constitutes a convenient tool of control.

If the M/RW ratio of a wheel is expressed in terms of the radial pressure distribution function $\sigma_{(\delta)}$ (Fig. 5) and the corresponding τ/σ function $t_{(\delta)}$, the following expression is obtained:

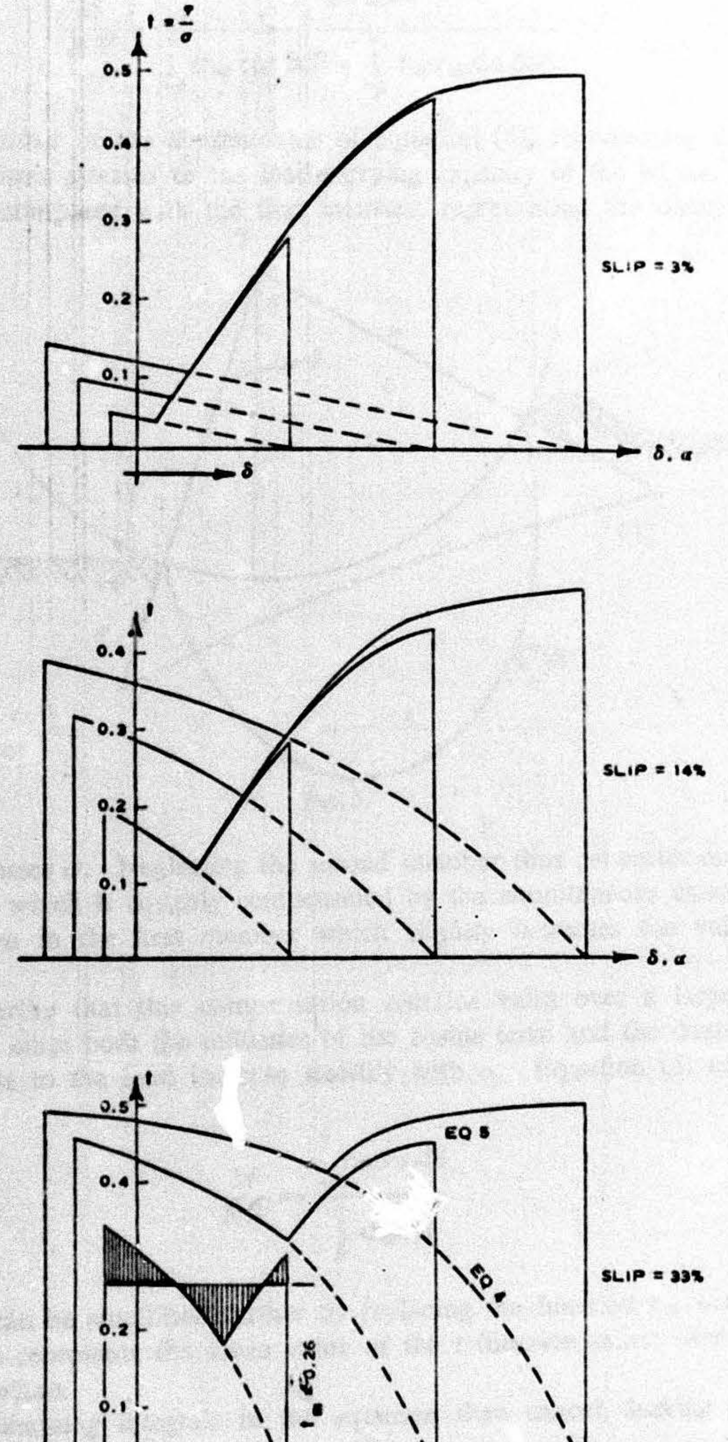
$t = \frac{\tau}{\sigma}$
0.5
0.4
0.3
0.2
0.1
SLIP = 3%
δ, α
δ
t
0.4
0.3
0.2
0.1
SLIP = 14%
δ, α
t
0.5
0.4
0.2
0.1
EQ 5
EQ 4
SLIP = 33%
$t_a = 0.26$
δ, α
β
δ
α = 0.3
α = 0.6
α = 0.9

FIG. 4.

$$\frac{M}{RW} = \frac{\int_{-\beta}^{\alpha} t_{(\delta)}\sigma_{(\delta)}\mathrm{d}\delta}{\int_{-\beta}^{\alpha} \sigma_{(\delta)} \cos\delta\mathrm{d}\delta + \int_{-\beta}^{\alpha} t_{(\delta)}\sigma_{(\delta)}\sin\delta\mathrm{d}\delta} \,. \tag{6}$$

The second member of the denominator of equation (6), representing the contribution of the shear stresses to the load-carrying capacity of the wheel, is a very small quantity compared with the first member, representing the contribution of

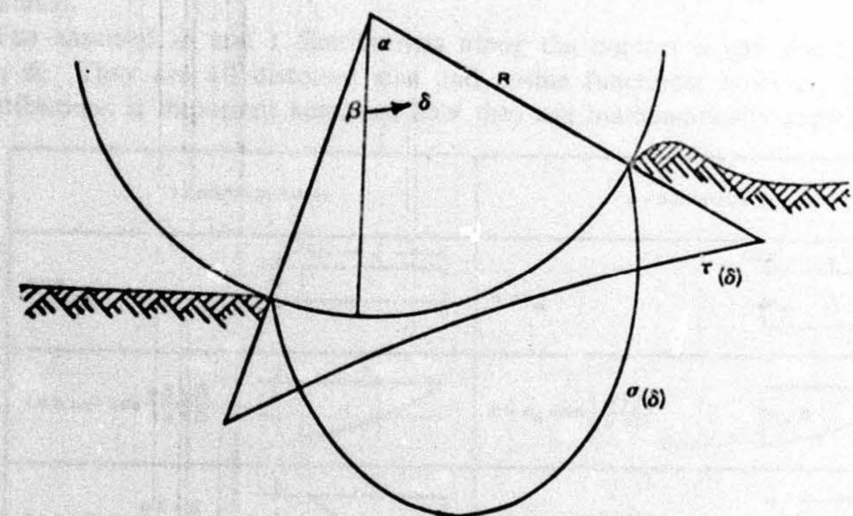

FIG. 5.

the normal stresses σ. Neglecting the second member thus generates only a small negative error, which is roughly compensated by the simultaneous cancellation of the cosine term in the first member which slightly increases the value of the denominator.

It is noteworthy that this compensation remains valid over a large range of contact angles, since both the influence of the cosine term and the contribution of the shear stress to the load increase steadily with α. Equation (6) can thus be rewritten

$$\frac{M}{RW} \approx \frac{\int_{-\beta}^{\alpha} t_{(\delta)}\sigma_{(\delta)}\mathrm{d}\delta}{\int_{\beta}^{\alpha} \sigma_{(\delta)}\mathrm{d}\delta} \,. \tag{7}$$

Equation (7) can be simplified further by replacing the function $t_{(\delta)}$ with the constant t_m which represents the mean value of the t function taken over the whole wheel–soil interface.

The two remaining integrals in the equation then cancel, leaving the simple approximation formula:

$$\frac{M}{RW} \approx t_m \tag{8}$$

This constitutes only a crude evaluation, but the approximate formula is remark-

able in its general validity since it can be developed without considering slip, wheel geometry, or soil strength. The inherent error depends only on the t and σ functions and on the integration limits α and β.

To check the degree of approximation, equation (8) was compared with the exact equation (6), which was solved by a computer for all possible combinations of five t distributions and four α functions. Assuming $\beta = 0{\cdot}2\alpha$, which is close to experimental values, M/RW was calculated for α varying from 0·1 to 1·1 (radians).

The assumed σ and t distributions along the contact length are tabulated in Fig. 6. They are all distorted sine and cosine functions; however, the type of distributions is important here, not how they are mathematically expressed. It is

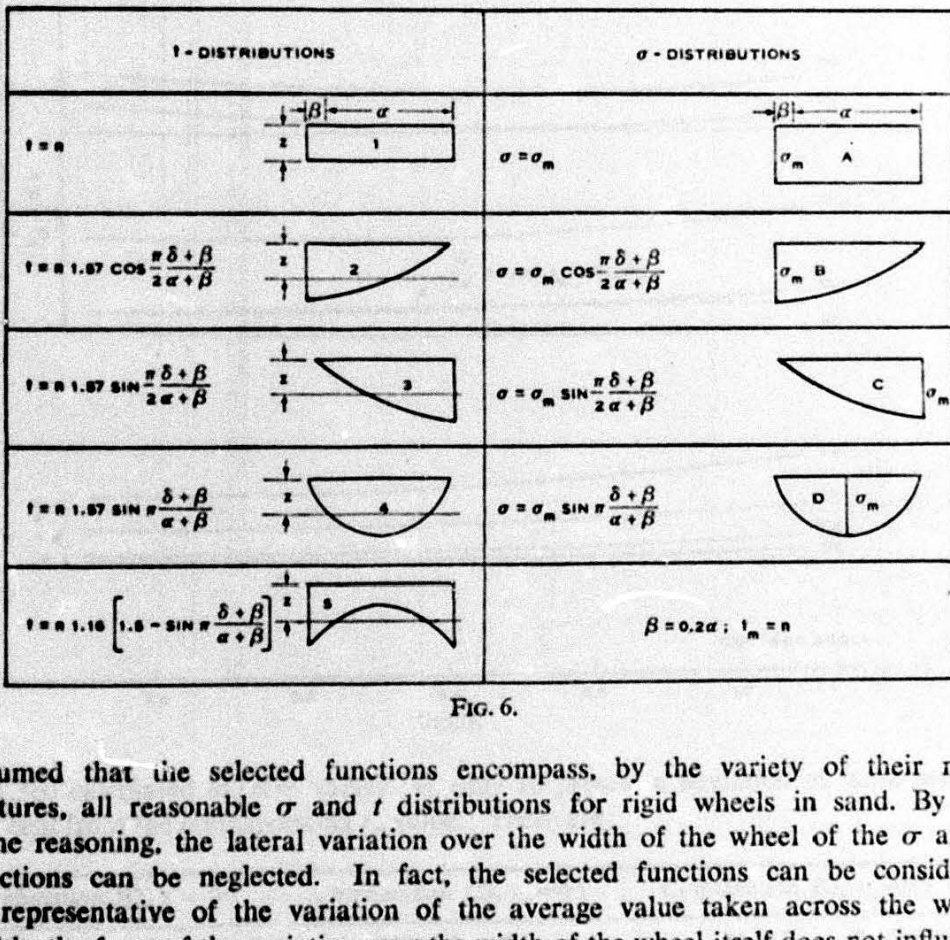

FIG. 6.

assumed that the selected functions encompass, by the variety of their main features, all reasonable σ and t distributions for rigid wheels in sand. By the same reasoning, the lateral variation over the width of the wheel of the σ and t functions can be neglected. In fact, the selected functions can be considered as representative of the variation of the average value taken across the wheel width; the form of the variation over the width of the wheel itself does not influence M/RW.

The t functions have been so formulated that their mean value t_m is equal to unity. By multiplying them with a constant, n, any desired t_m value can be introduced. Some of these results have been illustrated graphically in Fig. 7. The following were chosen:

(a) The M/RW curves with the greatest positive or negative deviation from the t_m value (combinations B2 or C3 and B3, respectively).

(b) The curve for the simplest σ–t combination (A1).

(c) The curve for the most probable σ–t combination among the investigated ones (combination D5).*

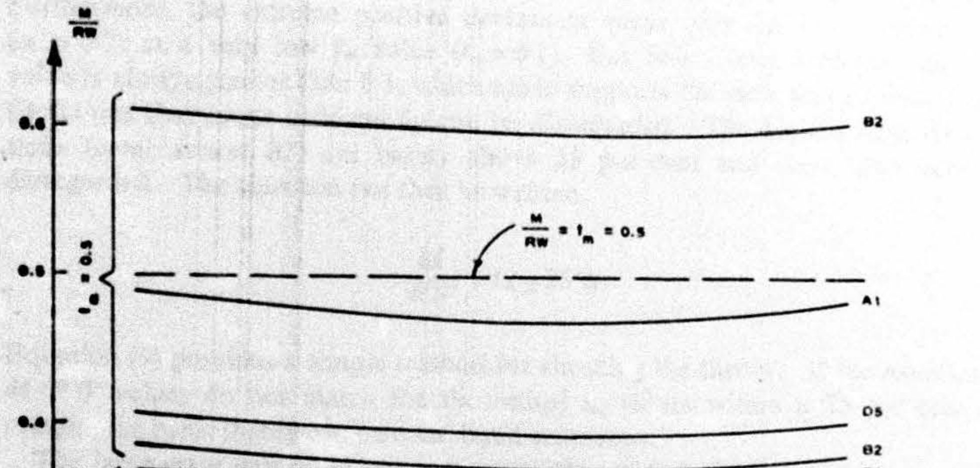

FIG. 7.

Of the 360 calculated M/RW values [4], only 10 present a deviation of more than 25 per cent from the corresponding t_m value. These are:

Combination B2, t_m=0·1			Combination C3, t_m=0·1			Combination B3, t_m=0·5		
α	M/RW	Deviation (%)	α	M/RW	Deviation (%)	α	M/RW	Deviation (%)
0·7	0·1258	+25·8	0·7	0·1288	+28·8	0·5	0·3702	−26.0
0·9	0·1281	+28·1	0·9	0·1354	+35.4	0·7	0·3657	−26·9
1·1	0·1312	+31·2	1·1	0·1451	+45.1	0·9	0·3634	−27·3
						1·1	0·3634	−27·3

*The t function 5 comes nearest to the distribution found by Sela. All measurements of σ distributions under rigid wheels on sand indicate a type of distribution comparable to D (Fig. 6.).

However, none of these combinations of extreme deviations is probable (C3 is the most improbable one of all those investigated), so they can be disregarded. Furthermore, the extreme positive deviations occur only for high contact angles ($\alpha > 0{\cdot}7$) at a very low t_m value ($t_m = 0{\cdot}1$). But Sela's tests indicate that the t_m value is always greater than 0·1, which again supports the view that extreme positive deviations that occur incidentally can be disregarded. The highest negative deviations (combination B3) are barely above 25 per cent and these also have been disregarded. The equation can then be written:

$$\frac{M}{RW} = t_m \pm 25\%. \tag{9}$$

Equation (9) provides a simple method for checking the theory. If the experimental M/RW values do not match the theoretical t_m values within a 25 per cent error margin, the basic theory can be considered erroneous.

The large error gap of 25 per cent exists almost exclusively because a number of σ–t combinations are considered jointly. Any individual curve for a given σ–t combination is remarkably stable and can be approximated with a relatively small error by a horizontal line of the equation.

$$\frac{M}{RW} \approx A_{\sigma t} t_m. \tag{10}$$

The subscripts σ and t indicate that the coefficient A is only valid for a particular σ–t combination, whatever this might be. For example, the coefficient $A_{D5} = 0{\cdot}825$ is appropriate for the most probable combination D5 (Fig. 7). The error is less than ± 5 per cent (if the extreme deviations of $+7$ and $+11{\cdot}5$ per cent at $\alpha = 0{\cdot}9$ and $\alpha = 1{\cdot}1$ for $t_m = 0{\cdot}1$ are again disregarded).

Since the value of $A_{\sigma t}$ is unknown for any real problem, equation (10) can be used only to compare the variations of t_m, not their absolute values. Because of the small error margin, equation (10) can be written:

$$\frac{M}{RW} \sim t_m. \tag{11}$$

This means that M/RW varies directly as t_m, if it can reasonably be assumed that the general aspect of the σ and t distributions does not change radically during the test program. This assumption is a basic requirement for the application of equation (11).

This requirement is met by a series of constant-slip tests in which the contact length is varied by increasing load. While one easily envisions basic changes in the σ and t distributions with varying slippage, these functions can be assumed to be essentially stable for constant-slip conditions.

In practical application, equation (11) could be used as a second means of control by describing both the experimental M/RW curves of constant-slip tests and the theoretical t_m curves in general qualitative terms. If the two sets of descriptive terms do not coincide, the evaluated theory is erroneous. While equation (9)

permits a quantitative comparison, equation (11) can only be used in a qualitative sense.

COMPARISON OF EXPERIMENTAL AND THEORETICAL RESULTS

Figure 8 shows the variation of the t_m values as functions of contact length drawn from Sela's theory. These curves were determined by graphical construction applied

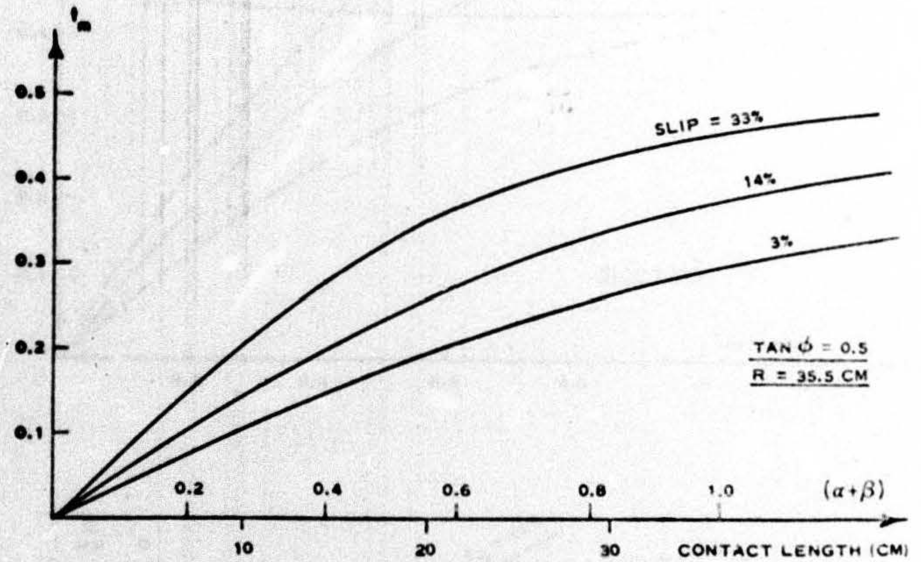

FIG. 8.

on the τ/σ variations represented in Fig. 4 (as shown for one example at the bottom of the figure). After application of the appropriate correction factor to account for the different tan ϕ value as explained earlier, these curves were superposed onto the corresponding experimental M/RW curves obtained from constant-slip, rigid wheel tests together with the ± 25 per cent error limits (Figs. 9, 10).

Pertinent data from these tests are:

G	Soil strength (N/cm^3)	tan ϕ	Wheel diameter (cm)	Wheel width (cm)	Wheel slip (%)	Wheel load (N)
5·44	60	0·65	71	7·6	3, 14, 33	178, 400, 1066, 2132, 3860, 5550
5·44	60	0·65	71	30·5	3, 14, 33	444, 1775, 3550
2·72	30	0·60	71	7·6	3, 14, 33	178, 378, 845, 1242, 1732
2·72	30	0·60	71	30·5	3, 14, 33	444, 888, 1776, 3552, 7105

The total contact length was measured directly by means of a sensor in the surface of the wheel. Test equipment and procedures used are described in reference [4].

Using equation (9) as a criterion, the experimental M/RW curves should remain inside the error limits (dashed lines in the figures), if Sela's theory is correct. Roughly 50 per cent of the experimental M/RW curves are beyond these limits, which seems sufficient to reject Sela's theory.

However, the close accordance of Sela's theoretical predictions to his measurements of the τ/σ distribution by means of tangential and normal load cells remains

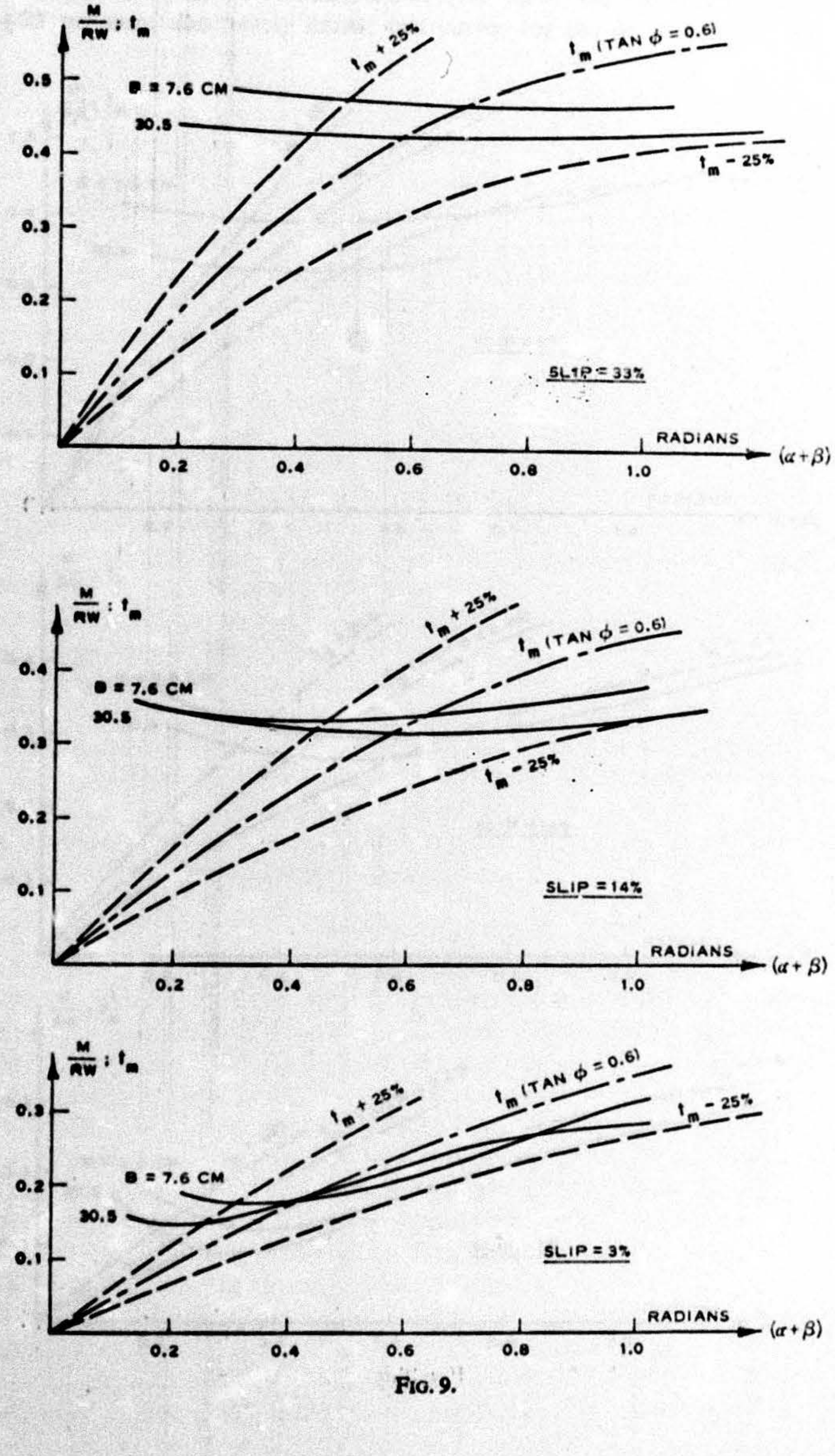

FIG. 9.

to be discussed. If one compares the results of tests on the weaker sand ($\tan\phi=0{\cdot}6$, Fig. 9) to those of the stronger sand ($\tan\phi=0{\cdot}65$, Fig. 10), it is seen that for high contact angles ($\alpha+\beta>0{\cdot}5$ radians), the M/RW curves for the weak soil are within the ± 25 per cent uncertainty limits; test curves for the dense soil are not. Pro-

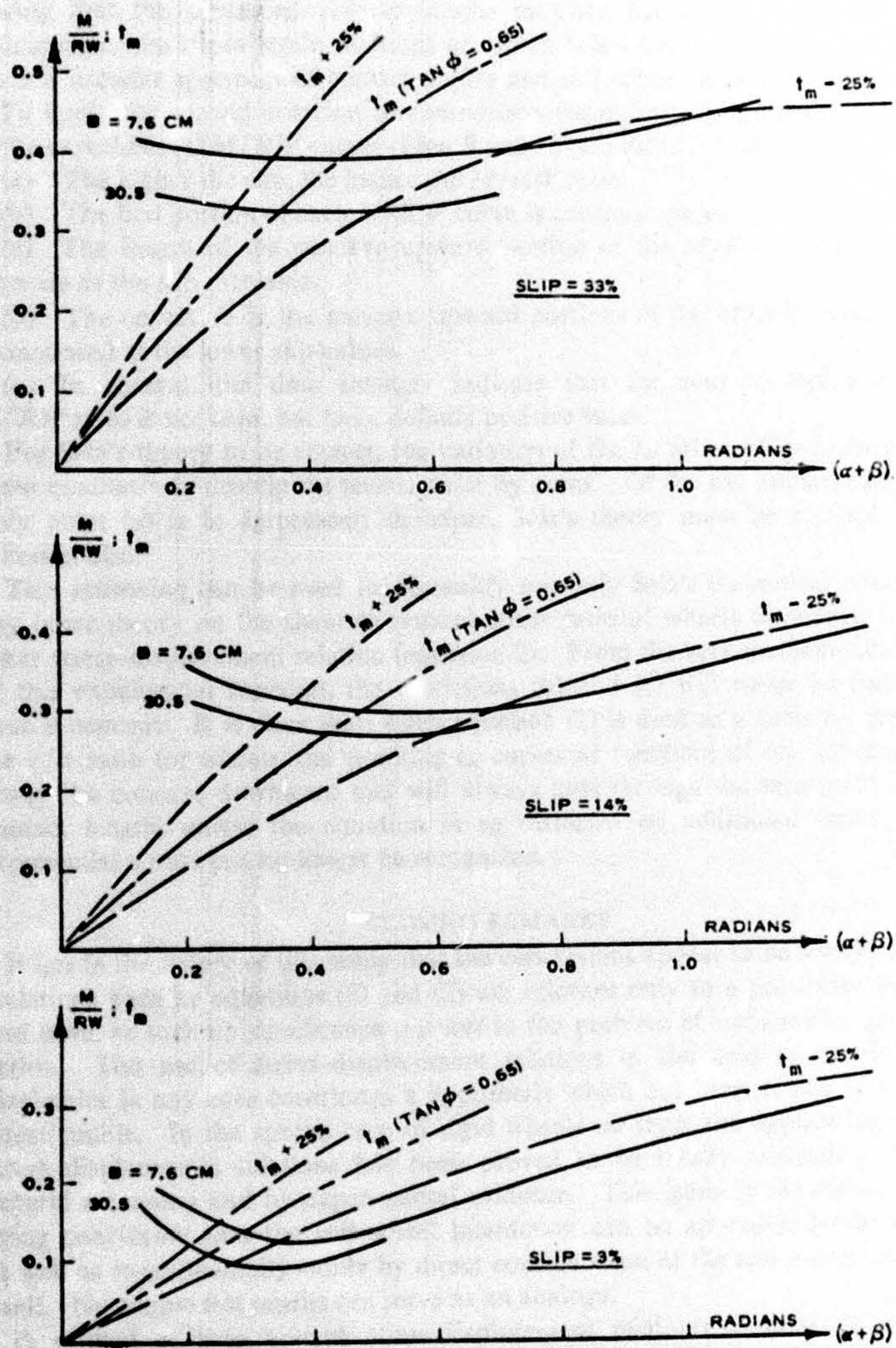

FIG. 10.

jecting these findings on the even weaker sand used by Sela ($\tan \phi = 0{\cdot}44$) and taking into account the fact that Sela's tests were limited to rather high contact angles (0·6 to 1·2 radians), it is concluded that Sela's theory coincidentally agrees closely with test results under these particular circumstances. Hence, it is not surprising that the measured τ/σ variations matched his theory fairly well. The deficiency of the stress–strain relations on which Sela's theory is based appears only when a broader spectrum of contact angles and soil strengths is considered.

To apply the second criterion of comparison (equation 11), the main descriptors of the experimental M/RW curves (Figs. 9 and 10) are listed below:

(a) The higher the slip, the higher the M/RW ratio.

(b) The first portion of each M/RW curve is concave upward.

(c) The length of the concave upward portion of the M/RW curve seems* to increase as the slip increases.

(d) The curvature in the concave upward portions of the M/RW curve is more pronounced at the lower slip values.

(e) In general, the data strongly indicate that for zero contact length, the M/RW ratio is not zero, but has a definite positive value.

For Sela's theory to be correct, the variation of the t_m values (Fig. 8) must match these qualitatively descriptive terms, point by point. Of the five enumerated points, only point (a) is in agreement; therefore, Sela's theory must be rejected by this criterion also.

This reasoning can be used to disqualify not only Sela's theoretical concept but any other theory on the shear to normal stress ratio of wheels developed from the shear stress–displacement relation (equation 2). From the very mathematical nature of this exponential function, the conditions (b) and (e) will never be fulfilled by such a concept. It is clear that, when equation (2) is used as a basis for predicting the τ/σ ratio for wheels, the resulting t_m curves as functions of contact length will always be concave downward and will always pass through the zero point for zero contact length, unless the equation is so distorted by additional terms that its exponential nature can no longer be recognized.

CLOSING REMARKS

It lies in the nature of this study that the conclusions appear to be solely negative: Relations such as equations (1) and (2) are relevant only to a particular test setup and have, as such no significance *a priori* to the problem of soil-running gear interaction. The use of stress–displacement relations in the field of terrain–vehicle mechanics in any case constitutes a hypothesis which has been shown to be highly questionable. In the special case of rigid wheels on sand, the application of shear stress-displacement relations has been proved to be totally misleading, both by general reasoning and by experimental evidence. This leads to the rather discouraging conclusion that the soil–wheel interaction can be approached—theoretically as well as experimentally—only by direct consideration of the soil–wheel interaction itself. No simple test results can serve as an analogy.

A critical attitude toward stress-displacement relations is certainly not new. Reece [14] appears to accept them only because "there seems to be no alternative

*Some curves were too short or too flat to recognize the end of the concave upward portion.

at the moment". However, the author's conclusion goes so far as to say that the use of such relations, regardless of their mathematical expression, is basically misleading and can no longer be considered as a permissible simplifying assumption. Alternatives will have to be found.

Thus, from a certain point of view, the conclusions drawn herein are neither negative nor discouraging: Clearing away obstacles is a prerequisite to a fresh unobstructed view.

Recent publications attest the fact that as far as experimental research is concerned, a new approach to the key problem of soil–wheel interaction is feasible. Besides the contributions already mentioned [8, 9], mention should be made of the papers of Yong and Osler [15] and of Boyd and Windisch [16] who visualized some aspects of the wheel–soil interplay by an X-ray technique.

In order to show that such experimental findings are suitable as a starting point for theoretical conceptions, the author has attempted to develop a new approach to the problem of the shear-to-normal stress ratio at the wheel–soil interface. This will be the subject of a subsequent paper.

NOTATION

- A factor of proportionality, dimensionless
- B wheel width, cm
- c cohesion, lb/in^2
- G penetration resistance gradient, N/cm^3
- j displacement or distortion, in.
- k horizontal soil deformation module, in.
- k_c cohesive modulus of sinkage, lb/in$^{(1+n)}$
- k_ϕ frictional modulus of sinkage, lb/in$^{(2+n)}$
- M torque, N cm
- m factor of proportionality in Sela's formula, dimensionless
- n sinkage exponent, dimensionless (multiplication constant, dimensionless)
- R wheel radius, cm
- s slip, %
- t shear to normal stress ratio, dimensionless
- t_m mean shear to normal stress ratio, dimensionless
- W wheel load, N
- x distance from the leading edge to a considered point on the wheel–soil interface, in.
- z sinkage, in.
- α angle at the wheel center between the leading edge of the wheel–soil interface and the vertical reference line, deg or radians
- α_s angle designating the separation point between bulldozing and trailing zone, according to Sela's theory, deg

β angle at the wheel center between the trailing edge of the wheel–soil interface and the vertical reference line, deg or radians

δ position angle, angular coordinate, radians

σ normal or radial stress, N/cm^2

τ tangential or shear stress, N/cm^2

ϕ angle of internal friction, deg

Note: The dimensions of quantities proper to this paper are given in the Metric System. Those pertaining to quoted expressions from the Anglo-Saxon Literature are given in the original form.

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13. ABSTRACT

Current theoretical concepts of the soil-running gear interaction are based on empirical pressure-sinkage and shear stress-displacement relations. It was found that the tests from which these relations were obtained present a poor analogy, at best, to the soil-running gear interaction. In particular, the use of the shear stress-displacement relation for an analytical evaluation of the shear stresses at the contact surface of rigid wheels on sand was found to be misleading, primarily because equations obtained by the bevameter or dragged plate tests describe the soil behavior in the intermediate state, which is a state between the elastic and plastic states. Recent publications point out that part of the soil in the immediate vicinity of a powered wheel is in a state of actual plastic flow and part is in a quasielastic state. Thus, the soil-bevameter interaction is of a fundamentally different nature than the soil-wheel interaction, which makes results of such tests unsuitable for predicting wheel performance. Sela's theory of the relation between a rigid wheel and dry sand is based exclusively on the shear stress-displacement concept, and thus provided an excellent means of checking the concept as a whole. Using a simple approximate relation between the M/RW ratio and the mean shear-to-normal stress ratio taken over the total contact surface, the theory was checked by means of constant-slip rigid wheel tests. The experimental findings strongly supported the conclusion that shear stress-displacement relations are irrelevant to soil-wheel mechanics. A new theory was developed to assess the variation of the shear-to-normal stress ratio along the soil-wheel interface, without referring to shear stress-displacement relations. This is a subject of another paper.

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